

ON S_5 -COMPACT SPACES

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ABSTRACT

The aim of this paper is to introduce and study the concepts of S_5 -compact space, S_5 -compact subspace and countably S_5 -compact space via S_5 -open sets like wise to investigate their relationships to other well known types of compactness.

KEYWORDS: S_5 -Open, S_5 -Compact, Countably S_5 -Compact

1. INTRODUCTION

Compactness plays an important role in general topology and so many authors have been discussed and introduced weak and strong forms of compactness. In 1981, semi-compactness was studied and investigated by Dorselt [10] and several new and interesting results have been found. In 1976, Thompson [29] defined the concept of S-closed spaces and gave several properties about this space. Mathur [21] established several properties and relations between compact and S-closed spaces. The class of countably S-closed space was introduced by Dłaska [5] Recently, the authors introduced the concept of S_5 -open and S_5 -continuity [16]. The main purpose of this paper to introduce the notation of S_5 -compact space. we give some characterizations of S_5 -compact spaces by using filters and investigate its relation with other types of compactness and define countably S_5 -compact. In the last section, we give important characterizations about S_5 -compact subspaces and S_5 -sets.

For a subset A of a space X , $cl(A)$ and $int(A)$ represent the closure and interior of A respectively. A subset A of X is called semi-open [19] (α -open [22], pre-open [20], regular open [28]) set if $A \subseteq cl(int(A))$, (resp., $A \subseteq int(cl(int(A)))$, $A \subseteq int(cl(A))$, $A = int(A)$). The complement of semi-open (α -open, pre-open, regular open) set is called semi-closed (resp., α -closed, pre-closed, regular closed) set. A subset A of topological space (X, τ) is called θ -open (resp., δ -open) set [30] if for each $x \in A$, there is an open (resp., open) set U such that $x \in U \subseteq cl(U) \subseteq A$ (resp., $x \in U \subseteq int(U) \subseteq A$). A subset A of a topological space X is said to be θ -semi-open [13] (resp., semi- θ -open [4]) set if for each $x \in A$, there is a semi-open set U such that $x \in U \subseteq cl(U) \subseteq A$ (resp., $x \in U \subseteq scl(U) \subseteq A$). A set A is called semi-regular [6], if it is both semi-open and semi-closed also. The family of all semi open (resp., regular open, pre-open, regular-semi-open, θ -open, θ -semi-open, semi- θ -open, semi-regular) sets of X is denoted by $SO(X)$ (resp., $RO(X)$, $PO(X)$, $RSO(X)$, $\theta O(X)$, $\theta SO(X)$, $S\theta O(X)$, $SR(X)$).

2. PRELIMANARIES

In this section, we give some definitions and results which are needed in the next sections.

Definition 2.1: A topological space X is called:

- Locally indiscrete [3], if every open set in X is closed.
- Extremally disconnected [6], if the closure of every open subset of X is open or the interior of every closed subset of X is closed.
- Semi- T_1 [1], If for every two distinct points x, y in X , there exist two semi open sets, one containing x but not y and the other containing y but not x .
- Semi- T_2 [1], If for every two distinct points x, y in X , there exist two disjoint semi open sets U and V such that $x \in U$ and $y \in V$.

Proposition 2.2 [3]: If X is a locally indiscrete space, then each semi-open subset of X is closed and hence each semi-closed subset of X is open.

Definition 2.3: A filter base $\mathcal{B}$ in a topological space (X, τ) is said to be rc-converge [13] (resp., $\mathcal{S}$ -converge [30] and θ -converge [25]) to $x \in X$ if for each regular closed (resp., open and open) subset U of X containing x , there exists $F \in \mathcal{B}$ such that $F \subseteq U$ (resp., $\text{int } cl(F) \subseteq U$ and $cl(F) \subseteq U$).

Definition 2.4: A filter base $\mathcal{B}$ in a topological space (X, τ) is said to be rc-accumulate [13] (resp., $\mathcal{S}$ -accumulate [30] and θ -accumulate [25]) to a point $x \in X$ if for each regular closed (resp., open and open) subset U of X containing x , there exists an $F \in \mathcal{B}$ such that $F \cap U \neq \emptyset$ (resp., $\text{int } cl(F) \cap U \neq \emptyset$ and $cl(F) \cap U \neq \emptyset$).

Definition 2.5 ([5]): A topological space (X, τ) is called semi regular, if for each semi-closed F and each $x \notin F$ there exist disjoint semi-open sets U and V such that $F \subseteq U$ and $x \in V$.

Definition 2.6 [1]: A topological space (X, τ) is called is S^{**} -regular if and only if for every point $x \in X$ and every semi-closed set F in X such that $x \notin F$, there exists disjoint open sets G and H such that $x \in G$ and $F \subseteq H$.

Theorem 2.7 [1]: A topological space (X, τ) is called is S^{**} -regular if and only if for every semi-open G containing x , there exists a semi open H such that $x \in H \subseteq cl(H) \subseteq G$.

Definition 2.8: A space X is called semi-compact [11] (resp., compact [26], nearly compact [8], mildly compact [27], strongly compact [11] and θ -compact [25]) if every semi-open (resp., open, regular open, clopen, pre-open and θ -open) cover of X admits a finite subcover.

Definition 2.9 [7]: A topological space (X, τ) is called strongly S-closed if every closed cover of X has finite subcover.

Definition 2.10: A topological space (X, τ) is said to be S-closed [29] (resp., s-closed [4], N-closed [23] and quasi H-closed [9]) if for every semi-open (resp., semi-open, open and open) cover $\{U_\lambda : \lambda \in \Delta\}$ of X there exists a finite subfamily Δ_0 of Δ such that $X = \bigcup_{\lambda \in \Delta_0} cl(U_\lambda)$ (resp., $X = \bigcup_{\lambda \in \Delta_0} scl(U_\lambda)$, $X = \bigcup_{\lambda \in \Delta_0} \text{int } cl(U_\lambda)$, $X = \bigcup_{\lambda \in \Delta_0} cl(U_\lambda)$).

Lemmas 2.11: The following properties are true:

- A space X is S -closed if and only if every regular closed cover of X has a finite subcover [21].
- A space X is s -closed if and only if every semi regular cover of X has a finite subcover [4].

Definition 2.12: A space X is called countably S -closed [5] (resp., semi-countably compact [6]) if every countable cover of X by regular closed (resp., semi-open) sets has finite subcover.

Definition 2.13 [24]: A space X is called feebly compact if every countable open cover of X has a finite subfamily such that the closure of whose members cover X .

Definition 2.14 [15]: A function $f: X \rightarrow Y$ is called θS -continuous if the inverse image of each open subset of Y is θ –semi open set in X .

Definition 2.15 [16]: A semi open subset A of a space X is called S_S –open if for each $x \in A$, there is a semi closed set F such that $x \in F \subseteq A$.

The family of all S_S –open subsets of the topological space (X, τ) is denoted by $S_2O(X)$.

Lemma 2.16 [5]: A space X is semi-regular if and only if for every point $x \in X$ and every semi-open G containing x , there exists a semi open H such that $x \in H \subseteq scl(H) \subseteq G$.

Proposition 2.17 [16]: Let (Y, τ_Y) be subspace of (X, τ) and $A \subseteq Y$. If A is S_S –open set of Y and Y is semi-regular set in X , then A is S_S –open set in X .

Proposition 2.18 [16]: Let (Y, τ_Y) be an α -open subspace of a space X . If $A \in S_2O(X, \tau)$ and $A \subseteq Y$, then $A \in S_2O(Y, \tau_Y)$.

Proposition 2.19 [16]: Let X be topological space, and $A, B \subseteq X$. If $A \in S_2O(X)$ and B is α -open and semi-closed then $A \cap B \in S_2O(X)$

Definition 2.20 [16]: A function $f: X \rightarrow Y$ is called S_S –continuous, if the inverse image of every open set in Y is an S_S –open set in X .

Proposition 2.21 [16]: If a space X is semi- T_1 , then $SO(X) = S_2O(X)$

Definition 2.22 [18]: A function $f: X \rightarrow Y$ is called contra S_S -continuous if $f^{-1}(U)$ is S_S -closed in X for each open set U in Y

Definition 2.23 [17]: A function $f: X \rightarrow Y$ is called almost S_S -continuous (resp., weakly S_S -continuous) if for each $x \in X$ and each open set H in Y containing $f(x)$ there is an S_S -open set G containing x such that $f(G) \subseteq int(cl(H))$ (resp., $f(G) \subseteq cl(H)$).

Proposition 2.24 [17]: For a function $f: X \rightarrow Y$, the following statements are equivalent:

- f is almost S_5 -continuous.
- For each $x \in X$ and each $\mathcal{G}$ -open set in Y containing $f(x)$, there is an S_5 -open set U in X containing x such that $f(U) \subseteq H$.
- The inverse image of each $\mathcal{G}$ -open set in Y is S_5 -open in X .

3. CHARACTERIZATIONS OF S_5 -COMPACT SPACES

Definition 3.1: A filter base $\mathcal{B}$ in a topological space (X, τ) S_5 -converges (resp., $S_5 - \theta$ -converges) to a point $x \in X$, if for every S_5 -open set U containing x , there exists an $F \in \mathcal{B}$ such that $F \subseteq U$ (resp., $F \subseteq Cl(U)$).

Definition 3.2: A filter base $\mathcal{B}$ in a topological space (X, τ) S_5 -accumulates (resp., $S_5 - \theta$ -accumulates) to a point $x \in X$, if $F \cap U \neq \emptyset$ (resp., $F \cap Cl(U) \neq \emptyset$) for every S_5 -open set U containing x and every $F \in \mathcal{B}$.

It is clear from the above definitions that, if a filter base S_5 -converges (resp., S_5 -accumulates) to a point in a topological space, then it is $S_5 - \theta$ converges (resp., $S_5 - \theta$ accumulates) to the same point.

In the following example, we see that a filter base is $S_5 - \theta$ convergent which is not S_5 -convergent and $S_5 - \theta$ -accumulates which is not S_5 -accumulates.

Example 3.3: Consider $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{c\}, \{a, d\}, \{a, c, d\}\}$ and let $\mathcal{B} = \{\{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$. Then $\mathcal{B}$ is $S_5 - \theta$ convergent to c but it is not S_5 -convergent to c , because $\{c\}$ is S_5 -open set containing c and there is no an $F \in \mathcal{B}$ such that $F \subseteq \{c\}$.

Example 3.4: Consider the space (X, τ) given in Example 3.3 and let $\mathcal{B} = \{\{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$. then $\mathcal{B}$ is $S_5 - \theta$ accumulates to the point a but it is not S_5 -accumulates to a , because $\{a, d\}$ is as S_5 -open set containing a and there is $F = \{b, c\} \in \mathcal{B}$, such that $\{a, d\} \cap F = \emptyset$.

Remark 3.5: If a filter base $\mathcal{B}$ is S_5 -convergent to a point $x \in X$ in topological space, then x is an S_5 -accumulation point of $\mathcal{B}$

The converse of the above remark is not true in general as it is shown in Example 3.3. that the filter base $\mathcal{B}$ S_5 -accumulates to the point c .

Theorem 3.6: Let $\mathcal{B}$ be a filter base on a space X . Then the following statements are equivalent:

- The filter base $\mathcal{B}$ is S_5 -accumulates to a point $x \in X$.
- There exists a filter $\mathcal{B}_1$ finer than $\mathcal{B}$ and it is S_5 -convergent to x .

Proof: Obvious

Proposition 3.7: If $\mathcal{B}$ is a maximal filter base on a space X . Then $\mathcal{B}$ is S_5 -accumulates to a point $x \in X$ if and only if $\mathcal{B}$ is S_5 -convergent to x .

Proof: Let $\mathcal{B}$ be a maximal filter base on a space X and S_5 -accumulates to a point $x \in X$, then by Theorem 3.6,

there exists a filter $\mathcal{B}_1$ finer than $\mathcal{B}$ and S_2 -converge to x . But $\mathcal{B}$ is maximal filter base. Thus $\mathcal{B}$ is S_2 -convergence to x . Conversely. Follows from the definition

Proposition 3.8: Let $\mathcal{B}$ be a filter base in topological space (X, τ) if any semi-closed E in X containing x , there exists an $F \in \mathcal{B}$ such that $F \cap E \neq \emptyset$, then $\mathcal{B} S_2$ -accumulates to a point x .

Proof: Let U be an S_2 -open containing x , then for each $x \in U$, there exists a semi-closed set E in X such that $x \in E \subseteq U$. By hypothesis there exists $F \in \mathcal{B}$ such that $F \cap E \neq \emptyset$ which implies that $F \cap U \neq \emptyset$. Hence $\mathcal{B} S_2$ -accumulates to a point x .

Proposition 3.9: Let $\mathcal{B}$ be a filter base in topological space (X, τ) if for every semi-closed set E in X containing x , there exists an $F \in \mathcal{B}$ such that $F \subseteq E$, then $\mathcal{B} S_2$ -converges to the point x .

Proof: Let U be an S_2 -open set containing x , then for each $x \in U$, there exists a semi-closed set E in X such that $x \in E \subseteq U$. By hypothesis, there exists an $F \in \mathcal{B}$ such that $F \subseteq E$ implies that $F \subseteq U$. Hence $\mathcal{B} S_2$ -converges to x .

Theorem 3.10: Let $\mathcal{B}$ be a filter base in topological space (X, τ) . If $\mathcal{B}$ is S_2 -convergent (resp., S_2 -accumulates) to a point $x \in X$, then $\mathcal{B}$ is rc-convergent (resp., rc-accumulates) to x .

Proof: Let $\mathcal{B}$ be a filter base in a space X and let U be any regular closed set containing x , then $U \in S_2O(X)$. Since $\mathcal{B}$ is S_2 -convergent (resp., S_2 -accumulates) to the point x , then there exist $F \in \mathcal{B}$ such that $F \subseteq U$ (resp., $F \cap U \neq \emptyset$) this implies that $\mathcal{B}$ is rc-convergent (resp., rc-accumulates) to x . The converse of Theorem 3.10 is not true in general as it is shown in following example:

Example 3.11: In Example 3.3. The family of regular closed sets are only $\emptyset$ and X and hence $\mathcal{B}$ is rc-convergent to c but it is not S_2 -convergent to c . And $\mathcal{B}$ is rc-accumulates to a but it is not S_2 -accumulate to a .

Proposition 3.12: Let $\mathcal{B}$ be a filter base in topological space (X, τ) . If $\mathcal{B}$ is S_2 -convergent (resp., S_2 -accumulates) to a point $x \in X$, then $\mathcal{B}$ is θ -convergent (resp., θ -accumulates) to x .

Proof. Similar to the proof of Theorem 3.10

Proposition 3.13: Let $\mathcal{B}$ be a filter base in topological space (X, τ) . If $\mathcal{B}$ is S_2 -convergence (resp., S_2 -accumulates) to a point $x \in X$, then $\mathcal{B}$ is δ -convergent (resp., δ -accumulates) to x .

Proof: Let $\mathcal{B}$ be a filter base in a space X and let U be any open set containing x , then $\text{int cl}(U)$ is regular open set in X . So, $\text{int cl}(U) \in S_2O(X)$ since $\mathcal{B}$ is S_2 -convergence (resp., S_2 -accumulates) to a point x , then there exist $F \in \mathcal{B}$ such that $F \subseteq \text{int cl}(U)$ (resp., $F \cap \text{int cl}(U) \neq \emptyset$) this implies that $\mathcal{B}$ is δ -convergence (resp., δ -accumulates).

Theorem 3.14: If $f: X \rightarrow Y$ is an S_2 -continuous (resp., almost S_2 -continuous) function, then for each point $x \in X$ and each filter base $\mathcal{B}$ on a space X which is S_2 -convergence to x , the filter base $f(\mathcal{B})$ is convergent (resp., δ -convergent) to $f(x)$.

Proof: Suppose that x belong to X and $\mathcal{B}$ is any filter base in X which S_2 -convergent to x . By S_2 -continuity

(resp., almost S_5 -continuity) of f , for any open set V in Y containing $f(x)$, there exist $U \in S_5\mathcal{O}(X)$ containing x such that $f(U) \subseteq V$ (resp., $f(U) \subseteq \text{int } cl(V)$). But $\mathcal{B}$ is S_5 -convergence to x in X , so there exist $F \in \mathcal{B}$ such that $F \subseteq U$ implies that $f(F) \subseteq V$ (resp., $f(F) \subseteq \text{int } cl(V)$). Thus $f(\mathcal{B})$ is convergent (resp., δ -convergent) to $f(x)$.

Definition 3.15: A topological space (X, τ) is called S_5 -compact if for every S_5 -open cover $\{U_\lambda : \lambda \in \Delta\}$ of X , there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\lambda \in \Delta_0} U_\lambda$. Since every S_5 -open set is semi-open, we obtain the following result.

Remark 3.16: Every semi-compact space is S_5 -compact.

The following example shows that the converse is not true.

Example 3.17: Let $X = \mathbb{R}$ and $\tau = \{\emptyset\} \cup \{R\} \cup \{(-\infty, b) : b \in \mathbb{R}\}$, then the family of semi-open sets is $SO(X) = \{\emptyset, R, \{(-\infty, b) : b \in \mathbb{R}\}, \{(-\infty, b] : b \in \mathbb{R}\}\}$ and the family of S_5 -open sets is $S_5\mathcal{O}(X) = \{\emptyset, R\}$. Hence X is S_5 -compact but it is not semi-compact.

Proposition 3.18 [11]: Let X be a semi T_2 -space then X is semi-compact if and only if it is finite.

Corollary 3.19: Let X be semi T_2 -space then X is S_5 -compact if and only if it is finite.

Proof: Straightforward.

Theorem 3.20: If a space X is S^{**} -regular and S_5 -compact then it is semi-compact.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be semi-open cover of X , then for each $x \in X$, there exist $\lambda_x \in \Delta$ such that $x \in U_{\lambda_x}$. Since X is S^{**} -regular, then by Theorem 2.8, there exists an open set V of X such that $x \in V \subseteq cl(V) \subseteq U_{\lambda_x}$. This implies that $U_{\lambda_x} \in S_5\mathcal{O}(X)$. and since X is S_5 -compact, then there exists a finite subset Δ_0 of Δ such that $X = \bigcup_{\lambda \in \Delta_0} U_{\lambda_x}$. This shows that X is semi-compact.

Theorem 3.21: If a space X is S_5 -compact then it is θ -compact.

Proof: Follows from the fact every θ -open is S_5 -open set.

Corollary 3.22: If a topological space (X, τ) is S_5 -compact, then it is quasi H-closed and hence mildly compact.

Proof: Follow from Theorem 3.21 and the fact that each nearly compact is quasi H-closed and hence it is mildly compact.

Theorem 3.23: If a space X is S_5 -compact, then it is s-closed.

Proof Let $\{U_\lambda : \lambda \in \Delta\}$ be any semi open cover of X , then $\{cl(U_\lambda) : \lambda \in \Delta\}$ is a regular closed cover of X implies that $\{cl(U_\lambda) : \lambda \in \Delta\}$ is S_5 -open cover of X and since X is S_5 -compact, then there is finite subset Δ_0 of Δ such that $X = \bigcup_{\lambda \in \Delta_0} cl(U_\lambda)$. This shows that X is s-closed.

Theorem 3.24: If a space X is S_5 -compact, then it is S-closed.

Proof: let $\{F_\lambda : \lambda \in \Delta\}$ be a regular closed cover of X , then $\{F_\lambda : \lambda \in \Delta\}$ is S_F -open cover of X . Since X is S_F -compact, then there exist finite subset Δ_* of Δ such that $X = \bigcup_{\lambda \in \Delta_*} F_\lambda$ and this shows that X is S -closed.

Theorem 3.25: For any topological space (X, τ) the following statements are equivalent:

- X is S_F -compact
- Every maximal filter base $\mathcal{B}$ on a space X is S_F -convergence to a point $x \in X$.
- Every filter base $\mathcal{B}$ on a space X is S_F -accumulates to some point $x \in X$.
- For every family $\{V_\lambda : \lambda \in \Delta\}$ of S_F -closed subsets of X , such that $\bigcap \{V_\lambda : \lambda \in \Delta\} = \emptyset$, there exist finite subset Δ_* of Δ such that $\bigcap \{V_\lambda : \lambda \in \Delta_*\} = \emptyset$.

Proof (1) $\Rightarrow$ (2): Suppose that X is S_F -compact and let $\mathcal{B} = \{F_\lambda : \lambda \in \Delta\}$ be a maximal filter base on X . suppose that $\mathcal{B}$ does not S_F -converges to any point of X . Since $\mathcal{B}$ is maximal filter base then by Proposition 3.7, $\mathcal{B}$ does not S_F -accumulates to any point of X . This implies that for every $x \in X$, there exists an S_F -open set U_x and $F_{\lambda(x)} \in \mathcal{B}$ such that $U_x \cap F_{\lambda(x)} = \emptyset$. The family $\{U_x : x \in X\}$ is an S_F -open cover of X and by (1), there exist finite number of points $x_1, x_2, x_3, \dots, x_n$ such that $X = \bigcup \{U_{x_i} : i = 1, 2, 3, \dots, n\}$. Since $\mathcal{B}$ is a filter base on X , there exists $F \in \mathcal{B}$ such that $F \subseteq \bigcap \{F_{\lambda(x_i)} : i = 1, 2, \dots, n\}$. Hence $F \cap U_{x_i} = \emptyset$ for each $i = 1, 2, \dots, n$ implies that $\bigcap \{U_{x_i} : i = 1, 2, 3, \dots, n\} = \emptyset$, then $F \cap X = \emptyset$. Therefore, $F = \emptyset$ which contradicts the fact that $F \neq \emptyset$. Thus $\mathcal{B}$ is S_F -convergence to a point x .

(2) $\Rightarrow$ (3). Let $\mathcal{B}$ be any filter base on X , then there exists a maximal filter base $\mathcal{B}_1$ such that $\mathcal{B} \subseteq \mathcal{B}_1$. By (2), $\mathcal{B}_1$ is S_F -convergent to some point $x \in X$, this implies that for every S_F -open set U containing x , there exists $F_1 \in \mathcal{B}_1$ such that $F_1 \subseteq U$. Thus, for every $F \in \mathcal{B}$, we have $\emptyset \neq F \cap F_1 \subseteq F \cap U$ this shows that $\mathcal{B}$ is S_F -accumulates to the point x .

(3) $\Rightarrow$ (4). Let $\{V_\lambda : \lambda \in \Delta\}$ be family of S_F -closed subsets of X such that $\bigcap \{V_\lambda : \lambda \in \Delta\} = \emptyset$. Suppose that every finite subfamily $\bigcap \{V_{\lambda_i} : i = 1, 2, \dots, n\} \neq \emptyset$. Therefore, $\mathcal{B} = \left\{ \bigcap_{i=1}^n V_{\lambda_i} : n \in \mathbb{N}, V_{\lambda_i} \in \{V_\lambda : \lambda \in \Delta\} \right\}$ forms a filter base on X . Then by (3), $\mathcal{B}$ is S_F -accumulates to some points $x \in X$ which implies that for every S_F -open set U containing x , $V_\lambda \cap U \neq \emptyset$ for every $V_\lambda \in \mathcal{B}$ and each $\lambda \in \Delta$. Since $x \in \bigcap \{V_\lambda : \lambda \in \Delta\}$, then there exist $\lambda_* \in \Delta$ such that $x \in V_{\lambda_*}$. Therefore $x \in X \setminus V_{\lambda_*}$ which is S_F -open set in X . But $V_{\lambda_*} \cap X \setminus V_{\lambda_*} = \emptyset$ which is a contradiction since S_F -accumulates to x . And this completes the proof.

(4) $\Rightarrow$ (1). Let $\{U_\lambda : \lambda \in \Delta\}$ be any S_F -open cover of X , then $\{X \setminus U_\lambda : \lambda \in \Delta\}$ is a family of S_F -closed subsets of X such that $\bigcap \{X \setminus U_\lambda : \lambda \in \Delta\} = \emptyset$. So, by (4), there exists a finite subset Δ_* of Δ such that this implies that $\bigcap \{X \setminus U_\lambda : \lambda \in \Delta_*\} = \emptyset$. Hence, $X = \bigcup \{U_\lambda : \lambda \in \Delta_*\}$. This shows that X is S_F -compact.

Proposition 3.26: If every semi-closed cover of X has finite subcover, then X is S_F -compact.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of X by S_F -open set, then for each $x \in X$, there exists a semi-closed set F_{λ_x} such that $x \in F_{\lambda_x} \subseteq U_{\lambda_x}$, thus $\{F_\lambda : \lambda \in \Delta\}$ be a semi-closed cover of X . By hypothesis, there exists a finite subset Δ_* of Δ such that $X \subseteq \bigcup_{\lambda \in \Delta_*} F_\lambda \subseteq \bigcup_{\lambda \in \Delta_*} U_\lambda$. Hence X is S_F -compact.

Proposition 3.27: If X is S_5 -compact and semi-regular space, then it is compact.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be an open cover of X . Since X is semi-regular then by Lemma 2.16, $U_\lambda \in S_5\mathcal{O}(X)$ for every $\lambda \in \Delta$. Hence, $\{U_\lambda : \lambda \in \Delta\}$ is an S_5 -open cover of X and since X is S_5 -compact, then there exists a finite subset Δ_* of Δ such that $X = \bigcup_{\lambda \in \Delta_*} U_\lambda$. Hence X is compact.

Lemma 3.28: If a topological space (X, τ) is locally indiscrete, then $\tau = S_5\mathcal{O}(X)$.

Proof: Suppose that X is locally indiscrete and let U be an open subset of X . By definition of locally indiscrete $\text{int cl}(U) = U$, therefore U is regular open and hence U is S_5 -open. on the other hand let V be an S_5 -open set in X , then for each $x \in V$ there exist semi-closed set F such that $x \in F \subseteq V$ by proposition 2.2, F is an open subset of X this implies that V is an open set in X and this complete the proof.

Proposition 3.29: If a space X is locally indiscrete then X is S_5 -compact if and only if it is compact.

Proof: Follows from Lemma 3.28.

Proposition 3.30: Let $f: X \rightarrow Y$ be contra S_5 -continuous, surjective function if X is S_5 -compact, then Y is strongly S-closed.

Proof: Let $\{V_\lambda : \lambda \in \Delta\}$ be a closed cover of Y . Since f is contra S_5 -continuous, then $\{f^{-1}(V_\lambda) : \lambda \in \Delta\}$ is a cover of X by S_5 -open sets. Since X is S_5 -compact, then there exists a finite subset Δ_* of Δ such that $X = \bigcup_{\lambda \in \Delta_*} f^{-1}(V_\lambda)$ implies that $Y = f(X) = f(\bigcup_{\lambda \in \Delta_*} f^{-1}(V_\lambda)) = \bigcup_{\lambda \in \Delta_*} V_\lambda$. Hence Y is strongly S-closed.

Proposition 3.31: Let $f: X \rightarrow Y$ be S_5 -continuous (resp., almost S_5 -continuous) surjective function if X is S_5 -compact, then Y is compact (resp., δ -compact).

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be cover of X by open (resp., δ -open) sets. Since f is S_5 -continuous (resp., almost S_5 -continuous), then by Proposition 2.24, $\{f^{-1}(U_\lambda) : \lambda \in \Delta\}$ is cover of X by S_5 -open sets. Since X is S_5 -compact, then there exists a finite subset Δ_* of Δ such that $X = \bigcup_{\lambda \in \Delta_*} f^{-1}(U_\lambda)$ implies that $Y = f(X) = f(\bigcup_{\lambda \in \Delta_*} f^{-1}(U_\lambda)) = \bigcup_{\lambda \in \Delta_*} U_\lambda$. Hence Y is compact (resp., δ -compact).

Definition 3.32: A topological space (X, τ) is called countably S_5 -compact if every countable cover of X by S_5 -open sets has finite subcover.

Proposition 3.33: Every countably S_5 -compact space is countably S-closed.

Proof: Let $\{U_n : n \in \mathbb{N}\}$ be countable cover of a space X by regular closed sets, then $\{U_n : n \in \mathbb{N}\}$ is countable cover of a space X by S_5 -open sets and since X is countably S_5 -compact, then there exists a finite subset $\mathbb{N}_*$ such that $X = \bigcup_{n \in \mathbb{N}_*} U_n$. Hence X is countably S-closed.

Proposition 3.34: Every countably S_5 -compact space is feebly compact.

Proof: Let $\{U_n : n \in \mathbb{N}\}$ be countable cover of a space X by an open sets. Then $cl(U_n)$ is regular closed for each $n \in \mathbb{N}$ and hence $\{cl(U_n) : n \in \mathbb{N}\}$ is a countable cover of the space X by S_5 -open sets. Since X is countably S_5 -compact, then there exists a finite subset N_ϵ of $\mathbb{N}$ such that $X = \bigcup_{n \in N_\epsilon} cl(U_n)$ and this shows that X is feebly compact.

Proposition 3.35: In locally indiscrete space, every feebly compact is countably S_5 -compact.

Proof: Let $\{U_n : n \in \mathbb{N}\}$ be a countable cover of a space X by S_5 -open sets. Since X is locally indiscrete then by Lemma 3.28, U_n is an open set for each $n \in \mathbb{N}$ and hence $X = \bigcup_{n \in N_\epsilon} cl(U_n) = \bigcup_{n \in N_\epsilon} U_n$ and this complete the proof.

Remark 3.36: The following statements are true

- Every semi-countably compact space is countably S_5 -compact.
- Every S_5 -compact space is countably S_5 -compact.

Proposition 3.37: If a topological space (X, τ) is locally indiscrete then the following are equivalent:

- Countably compact.
- Countably S_5 -compact.

Proof: Obvious by Lemma 3.28

Definition 3.38 [2]: A space (X, τ) is called *km-perfect* if for every $G \in RO(X)$ and for every point $x \in X \setminus G$, there exists a sequence $\{U_n : n \in \mathbb{N}\}$ of open sets of X such that $\bigcup_{n \in \mathbb{N}} U_n \subseteq G \subseteq \bigcup_{n \in \mathbb{N}} cl(U_n)$ and $x \notin \bigcup_{n \in \mathbb{N}} cl(U_n)$.

Proposition 3.39 [2]: If (X, τ) countably S-closed and *km-perfect*, then it is extremally disconnected.

Corollary 3.40: If (X, τ) countably S_5 -compact and km-perfect, then it is extremally disconnected.

Proof: Follows from Proposition 3.39 and the fact every countably S_5 -compact space is countably S-closed.

Lemma 3.41 [2]: A subset A of a space (X, τ) is β -open if and only if $cl(A)$ is regular closed

Proposition 3.42: If a topological space (X, τ) is countably S_5 -compact space, then the following statements are true:

- For every countable cover $\{F_n : n \in \mathbb{N}\}$ by semi-open sets there exists a finite subset N_ϵ of $\mathbb{N}$ such that $X = \bigcup_{n \in N_\epsilon} cl(F_n)$.
- For every countable cover $\{H_n : n \in \mathbb{N}\}$ by β -open sets there exists a finite subset N_ϵ of $\mathbb{N}$ such that $X = \bigcup_{n \in N_\epsilon} cl(H_n)$.

Proof (1): Follows from the fact that the closure of semi-open is S_5 -open set.

(2) Follows from Lemma 3.45.

Proposition 3.43: If a space (X, τ) is countably S_5 -compact and A is semi regular subset of X , then (A, τ_A) is countably S_5 -compact.

Proof: Let $\{F_n : n \in \mathbb{N}\}$ be countable cover of A by S_5 -open sets in A . Since A is semi regular, then by Proposition 2.17, $\{F_n : n \in \mathbb{N}\}$ is a countable cover of A by S_5 -open sets in X . Therefore, $\{F_n : n \in \mathbb{N}\} \cup F^c$ is a countable cover by S_5 -open sets in X . Since X is countably S_5 -compact, then there exists a finite subset N_c of $\mathbb{N}$ such that $X = \bigcup_{n \in N_c} F_n \cup F^c$ and hence $A = \bigcup_{n \in N_c} F_n$. This shows that (A, τ_A) is countably S_5 -compact.

4. S_5 -SETS AND S_5 -COMPACT SUBSPACES

In this section, we investigate subsets of a topological space which are S_5 -compact relative to the space and also as a subspace.

Definition 4.1: A subset A of a space X is said to be an S_5 -set (resp., S_5 -compact subspace) if for every cover $\{U_\lambda : \lambda \in \Delta\}$ of A by S_5 -open subsets of X (resp., S_5 -open subsets in A), there exist finite subset Δ_c of Δ such that $A \subseteq \bigcup_{\lambda \in \Delta_c} U_\lambda$ (resp., $A = \bigcup_{\lambda \in \Delta_c} U_\lambda$).

Proposition 4.2: If A is semi regular subset of a space X which is also S_5 -set then A is an S_5 -compact subspace

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of A by S_5 -open subsets of A . Since A is semi regular, then by Proposition 2.17, $U_\lambda \in S_5\mathcal{O}(X)$ for each $\lambda \in \Delta$. Since A is S_5 -set, then there exist finite subset Δ_c of Δ such that $A \subseteq \bigcup_{\lambda \in \Delta_c} U_\lambda$. But $U_\lambda \subseteq A$ for each $\lambda \in \Delta$ and hence $A = \bigcup_{\lambda \in \Delta_c} U_\lambda$. This shows that A is an S_5 -compact subspace.

Proposition 4.3: An α -open, semi-closed subset A of a space X is S_5 -set if A is S_5 -compact subspace.

Proof: Let A be S_5 -compact subspace and let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of A by S_5 -open subsets of X , then by Proposition 2.19, $A \cap U_\lambda \in S_5\mathcal{O}(X)$ for each $\lambda \in \Delta$. Since A is α -open, then by Proposition 2.18, $A \cap U_\lambda \in S_5\mathcal{O}(A)$ for each $\lambda \in \Delta$. This implies that $\{A \cap U_\lambda : \lambda \in \Delta\}$ is S_5 -open cover of A . Since A is a S_5 -compact subspace, then there exists a finite subset Δ_c of Δ such that $A = \bigcup_{\lambda \in \Delta_c} (A \cap U_\lambda) \subseteq \bigcup_{\lambda \in \Delta_c} U_\lambda$. This shows that A is S_5 -set.

Proposition 4.4: A space X is S_5 -compact if and only if every proper S_5 -closed subset of X is S_5 -set.

Proof: Let A be a proper S_5 -closed subset of the S_5 -compact space X and let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of A by S_5 -open subsets of X . Since A is S_5 -closed, then $X \setminus A$ is S_5 -open. But $A \subseteq \bigcup_{\lambda \in \Delta} U_\lambda$ thus $X = \bigcup_{\lambda \in \Delta} U_\lambda \cup (X \setminus A)$. Since X is S_5 -compact, then there exists a finite subset Δ_c of Δ such that $X = \bigcup_{\lambda \in \Delta_c} U_\lambda \cup (X \setminus A)$. This implies that $A \subseteq \bigcup_{\lambda \in \Delta_c} U_\lambda$. Hence A is S_5 -set.

Conversely. Let $\{F_\lambda : \lambda \in \Delta\}$ be an S_5 -open cover of X . Thus $H = F_{\lambda_c}^c$ is an S_5 -closed subset of X for some $\lambda_c \in \Delta$ and $\{F_\lambda : \lambda \in \Delta\}$ is S_5 -open cover of H for each $\lambda \in \Delta - \{\lambda_c\}$. By hypothesis H is S_5 -set, then there exists a finite subset Δ_c of $\Delta - \{\lambda_c\}$ such that $H \subseteq \bigcup_{\lambda \in \Delta_c} F_\lambda$ and since $X = H \cup F_{\lambda_c}$, then $X = \bigcup_{\lambda \in \Delta_c} F_\lambda \cup F_{\lambda_c}$. Hence X is S_5 -compact.

Corollary 4.5: Every semi regular subset of an S_5 -compact space is an S_5 -compact subspace.

Proof: Let A be semi regular subset of the S_5 -compact space X . Therefore, A is S_5 -closed then by Proposition 4.4, A is S_5 -set and then by Proposition 4.2, A is S_5 -compact subspace.

Proposition 4.6: Every S_F -set in a space X is quasi H-closed.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of A by an open sets in X , then $cl(U_\lambda)$ is regular closed sets in X for each $\lambda \in \Delta$ which implies that $\{cl(U_\lambda) : \lambda \in \Delta\}$ is a cover of A by S_F -open sets in X . But A is S_F -set, then there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup_{\lambda \in \Delta_0} cl(U_\lambda)$. This shows that A is quasi H-closed.

Corollary 4.7: Let Y be any clopen subspace of a space X and A be any subset of Y then A is S_F -set of X if and only if A is S_F -set of Y .

Proof: Follows from Theorem 4.2 and Proposition 4.3.

Proposition 4.8: Every S_F -set in a space X is N-closed.

Proof: Similar to the proof of Proposition 4.6.

Corollary 4.9: In a semi T_1 -space, a subset A is a S_F -set in X if and only if it is semi-compact.

Proof: Follows from Proposition 2.21

Proposition 4.10: If A and B are two S_F -sets in a space X , then so is $A \cup B$.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of $A \cup B$ by S_F -open sets in X . Therefore $A \cup B \subseteq \bigcup_{\lambda \in \Delta} U_\lambda$ this implies that $A \subseteq \bigcup_{\lambda \in \Delta} U_\lambda$ and $B \subseteq \bigcup_{\lambda \in \Delta} U_\lambda$. Since A and B are S_F -sets in a space X , then there exist finite subsets M and N of Δ such that $A \subseteq \bigcup_{\lambda \in M} U_\lambda$ and $B \subseteq \bigcup_{\lambda \in N} U_\lambda$. This implies that $A \cup B \subseteq \bigcup_{\lambda \in M \cup N} U_\lambda$. Hence $A \cup B$ is an S_F -set.

Theorem 4.11: Let A and B be two subsets of a space X . If A is an S_F -set and B is S_F -closed set in X , then $A \cap B$ is an S_F -set in X .

Proof: Let $\{F_\lambda : \lambda \in \Delta\}$ be any cover of $A \cap B$ by S_F -open sets of X . Since B is S_F -closed set, then $X \setminus B$ is S_F -open, and thus $A \subseteq \bigcup_{\lambda \in \Delta} F_\lambda \cup (X \setminus B)$. But A is an S_F -set in X , then there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup_{\lambda \in \Delta_0} F_\lambda \cup (X \setminus B)$. Therefore $A \cap B \subseteq \bigcup_{\lambda \in \Delta_0} F_\lambda$. Hence $A \cap B$ is an S_F -set in X .

Corollary 4.12: Let A be an S_F -set in the space X and B is S_F -closed subset of X , then the following statements are true:

- If $B \subseteq A$, then B is an S_F -set in X .
- If A is closed and X is extremally disconnected, then $int(A)$ is S_F -set.

Proof: (1) Since A is an S_F -set and B is S_F -closed subset of X , then by Theorem 4.11, $A \cap B = B$ is S_F -set in X .

(2) Since A is closed, then $int(A)$ is regular open. Since X is extremally disconnected therefore, $int(A)$ is regular closed subset of X and hence $int(A)$ is S_F -closed subset of X , so by Theorem 4.11, $int(A)$ is S_F -set.

Corollary 4.13: Let X be an S_F -compact and extremally disconnected and A is closed subset of X . If the boundary of A , $b(A)$, is an S_F -set, then A is S_F -set in X .

Proof: If $b(A) = \emptyset$, then A is clopen subset of the S_F -compact space X and hence it is S_F -set in X . In case that

$b(A) \neq \emptyset$, A is closed set in the extremally disconnected space X , therefore $\text{int}(A)$ is a proper S_5 -closed subset of the S_5 -compact space X . Thus, by Proposition 4.4, $\text{int}(A)$ is an S_5 -set and hence by Proposition 4.10, A is S_5 -set in X .

Theorem 4.15: For any topological space (X, τ) , the following statements are equivalent:

- X is S_5 -set (resp., S_5 -compact subspace).
- Every maximal filter base $\mathcal{B}$ on a space X is S_5 -convergence to a point $x \in X$.
- Every filter base $\mathcal{B}$ on a space X is S_5 -accumulates to some point $x \in X$.
- For every family $\{V_\lambda : \lambda \in \Delta\}$ of S_5 -closed subsets of X , such that $\bigcap \{V_\lambda : \lambda \in \Delta\} = \emptyset$, there exists a finite subset Δ_0 of Δ such that $\bigcap \{V_\lambda : \lambda \in \Delta_0\} = \emptyset$.

Proof: Similar to the proof of Theorem 3.25

Proposition 4.16: Let A be a subset of topological space (X, τ) . If every cover of A by semi-closed subsets of X (resp., by semi-closed subsets of A) has a finite subcover, then A is S_5 -set (resp., S_5 -compact subspace).

Proof: Similar to the proof of Proposition 3.26.

proposition 4.17: Let $f: X \rightarrow Y$ be an S_5 -continuous function. If A is an S_5 -set in X , then $f(A)$ is compact.

Proof: Let $\{U_\lambda : \lambda \in \Delta\}$ be a cover of A by open sets in X . Since f is S_5 -continuous, then $f^{-1}(U_\lambda) \in S_5\mathcal{O}(X)$ for each $\lambda \in \Delta$. Since A is S_5 -set, then there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup_{\lambda \in \Delta_0} f^{-1}(U_\lambda)$ and thus $f(A) \subseteq f(\bigcup_{\lambda \in \Delta_0} f^{-1}(U_\lambda)) = \bigcup_{\lambda \in \Delta_0} U_\lambda$. Hence $f(A)$ is compact.

Corollary 4.18: If $f: X \rightarrow Y$ is θS -continuous and if A is S_5 -set, then $f(A)$ is compact.

Proof: Follows from the fact that each θS -continuous function is S_5 -continuous.

Corollary 4.19: If a function $f: X \rightarrow Y$ is S_5 -continuous (resp., almost S_5 -continuous) and A is S_5 -closed subset of the S_5 -compact space X , then $f(A)$ is S_5 -set (resp., N-closed)

Proof: Let $\{V_\lambda : \lambda \in \Delta\}$ be any cover of $f(A)$ by an open sets in Y , then for each $x \in A$, there exist $\lambda(x) \in \Delta$ such that $f(x) \in V_{\lambda(x)}$. Since f is S_5 -continuous (resp., almost S_5 -continuous), then there exists an S_5 -open set U_x such that $f(U_x) \subseteq V_{\lambda(x)}$ (resp., $f(U_x) \subseteq \text{int cl}(V_{\lambda(x)})$). Therefore, $\{U_x : x \in A\}$ is S_5 -open cover of A . Thus, there exists a finite subset Δ_0 of Δ such that $A \subseteq \bigcup \{U_x : x \in \Delta_0\}$ implies that $f(A) \subseteq f(\bigcup \{U_x : x \in \Delta_0\}) \subseteq \bigcup \{V_{\lambda(x)} : x \in \Delta_0\}$ (resp., $f(A) \subseteq f(\bigcup \{U_x : x \in \Delta_0\}) \subseteq \bigcup \{\text{int cl}(V_{\lambda(x)}) : x \in \Delta_0\}$) this shows that $f(A)$ is S_5 -set (resp., N-closed).

Definition 4.20: Let A be a subset of X . A function $f: X \rightarrow A$ is called weakly S_5 -continuous retraction if f is weakly S_5 -continuous and $f|_A: A \rightarrow A$ is the identity of A .

Proposition 4.21: Let A be a subset of X and $f: X \rightarrow A$ be a weakly S_5 -continuous retraction. If X is Urysohn and extremally disconnected space, then A is S_5 -closed set.

Proof: Suppose that A is not S_5 -closed set. Then there exists a point $x \in X$ such that $x \in S_5cl(A) \setminus A$. Since f is weakly S_5 -continuous retraction, we have $f(x) \neq x$. Since X is Urysohn, then there are two open sets U and V such that $x \in U$, $f(x) \in V$ and $cl(U) \cap cl(V) = \emptyset$. Hence, by Definition 2.23, there exists an S_5 -open set W containing x such that $cl(U) \cap W = \emptyset$. By Proposition 2.19, $cl(U) \cap W$ is an S_5 -open set containing x and hence $(cl(U) \cap W) \cap A \neq \emptyset$ because $x \in S_5cl(A)$. Therefore there exists a point $y \in (cl(U) \cap W) \cap A$. Since $y \in A$, we have $f(y) = y \in (cl(U) \cap W) \cap A \subseteq cl(U)$ and hence $f(y) \notin cl(V)$ this implies that $f(W) \subseteq cl(V)$ because $y \in W$. This contradicts the fact that f is weakly S_5 -continuous. Hence A is S_5 -closed set.

5. CONCLUSIONS

A new type of convergence of a filter base is defined by using the concept of S_5 -open sets this type of convergence is weaker than the s -convergence and stronger than rc -convergence and θ -convergence. By using this type of convergence the concept of S_5 -compactness is characterized and it is proved that the concept of S_5 -compactness is weaker than the s -compactness and stronger than the concept of quasi H -closed spaces and θ -compact spaces. Some other relations among this concept and other similar concepts were found. Moreover, the concepts of S_5 -open sets, S_5 -convergence of a filter base and S_5 -compactness can be extended to bitopological spaces.

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